QUESTION 1: (START A NEW PAGE)

1 (a) Find the value of e^{-2} correct to 2 decimal places.

2 (b) Factor $xe^{2x} - e^{x}$.

1 (c) Convert $\frac{2\pi}{5}$ radians to degrees.

2 (d) Find a primitive function of $4x + 1$.

(e) Write down the exact value of

1 (i) $\cos\frac{5\pi}{4}$.

1 (iii) $\sin^2\left(\frac{7\pi}{8}\right) + \cos^2\left(\frac{7\pi}{8}\right)$.

2 (f) Find the value of c if the point (3,0) lies on the curve $y = \log_e(x+c)$.

2 (g) Solve $\sin x = \frac{1}{2}$ for $0 \leq x \leq 2\pi$.

QUESTION 2: (START A NEW PAGE)

(a) Differentiate

2 (i) $\cos 3x$.

1 (ii) e^{3x+2}.

2 (iii) $\log_e(4x+3)$.

3 (b) Find the equation of the tangent to the curve $y = \log_e x$ at the point where $x = 1$.

(c) Find

2 (i) $\int x^3 + 4\sqrt{x}\, dx$.

2 (ii) $\int_0^4 \frac{1}{x + 2}\, dx$.

QUESTION 3: (START A NEW PAGE)

(a) Given the points P(5,2) , Q(–3,8) and R(11,10).

2 (i) Find the coordinates of S, the midpoint of QR.

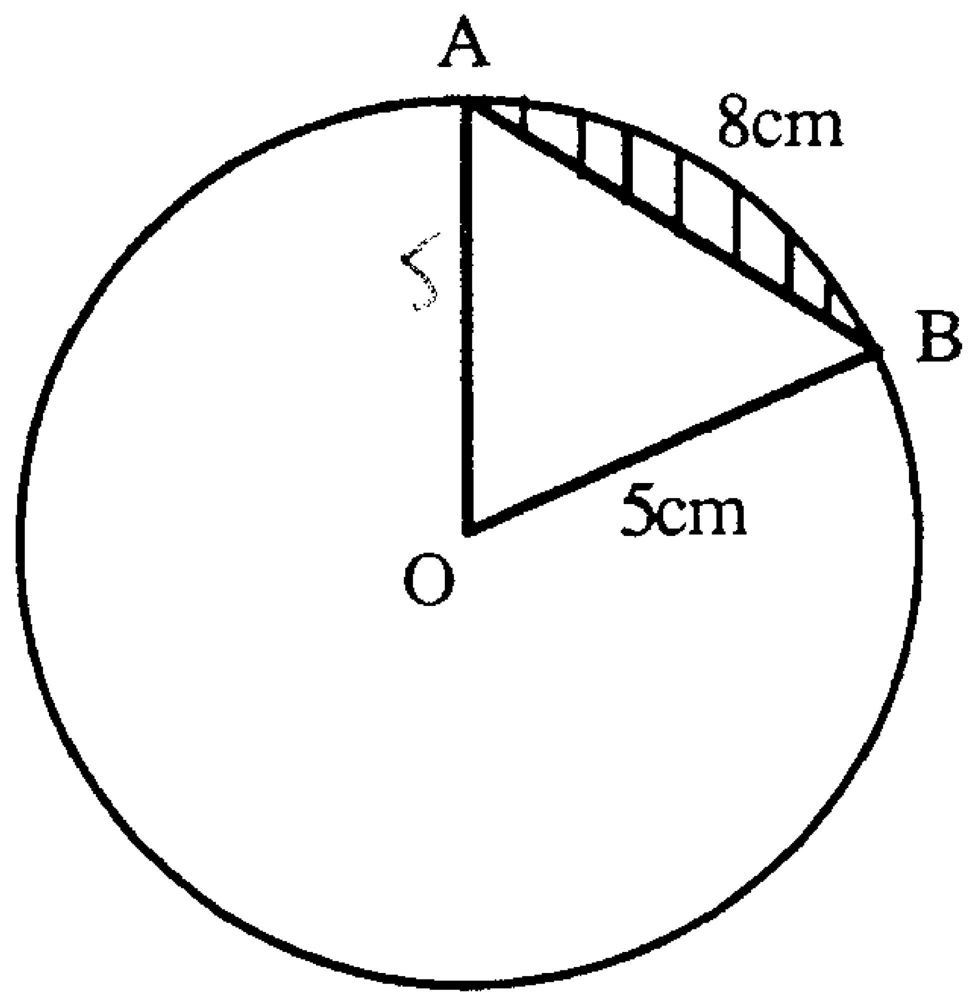
3 (ii) Show that SP is perpendicular to QR.

3 (iii) Hence, or otherwise, find the area of ΔPQR.

(b) In the circle with centre O, arc AB is 8cm and the radius OB is 5cm. Find

2 (i) the size of $\angle AOB$, giving your answer in radians.

2 (ii) the area of the shaded segment, giving your answer correct to 2 decimal places.



QUESTION 4: (START A NEW PAGE)

(a) ABCD is a parallelogram. M is the midpoint of BC and AM is drawn to meet DC produced at P.

1 (i) Draw a neat diagram of the above information.

3 (ii) Prove that ΔABM and ΔPCM are congruent.

2 (iii) Hence prove that C is the midpoint of DP.

(b) A boy tosses two regular six-sided dice and writes down the two numbers showing on the uppermost faces.

1 (i) Draw a dot diagram to illustrate the sample space.

Hence find the probability that

2 (ii) they show the same number,

1 (iii) their sum is an even number greater than 7,

2 (iv) their sum is an even number greater than 7, given that one of the numbers is odd.

QUESTION 5: (START A NEW PAGE)

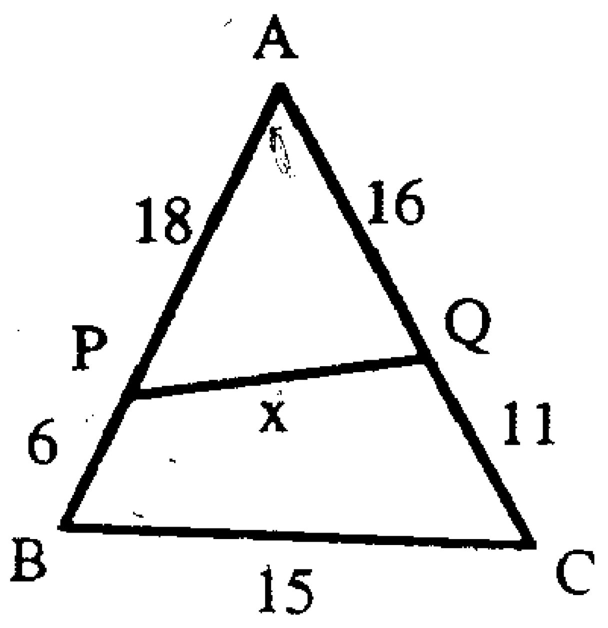
2 (a) (i) On the same set of axes sketch the curves $y = \cos x$ and $y = \cos 2x$ for $0 \le x \le \pi$.

2 (ii) Show that the curves intersect when $x = \frac{2\pi}{3}$.

3 (iii) Find the area bounded by these curves for $0 \le x \le \frac{2\pi}{3}$.

3 (b) (i) Prove that ΔABC and ΔAPQ are similar.

2 (ii) Find the value of x.



QUESTION 6: (START A NEW PAGE)

3 (a) (i) Find the co-ordinates of the two stationary points on the curve $y = x^3(x-4)$.

2 (ii) Determine the nature of the above stationary points.

1 (iii) Find the co-ordinates of all inflexion points.

2 (iv) Sketch the curve $y = x^3(x-4)$ for $-1 \le x \le 4$. Clearly indicate the position of all stationary and inflexion points and the intercepts with the co-ordinate axes.

(b) Measurements of the height of an irregular hole in a wall are made at regular intervals across the one metre wide hole. These values are used to find an approximation for the area of the hole.

Distance from left edge (cm)	0	25	50	75	100
Height of hole (cm)	23	32	37	24	17

2 (i) Use Simpson's Rule and all of the values from the above table to find an approximation for the area of the hole. Give your answer correct to the nearest 100cm^2.

2 (ii) Several measurements on sections of the hole indicate that there is an average airflow of 1250cm^3 per minute through any 10cm^2 section of the hole. Assuming that the airflow remains constant, calculate the approximate volume of air that would flow through the hole in one hour. Give your answer to the nearest cubic metre.

QUESTION 7: (START A NEW PAGE)

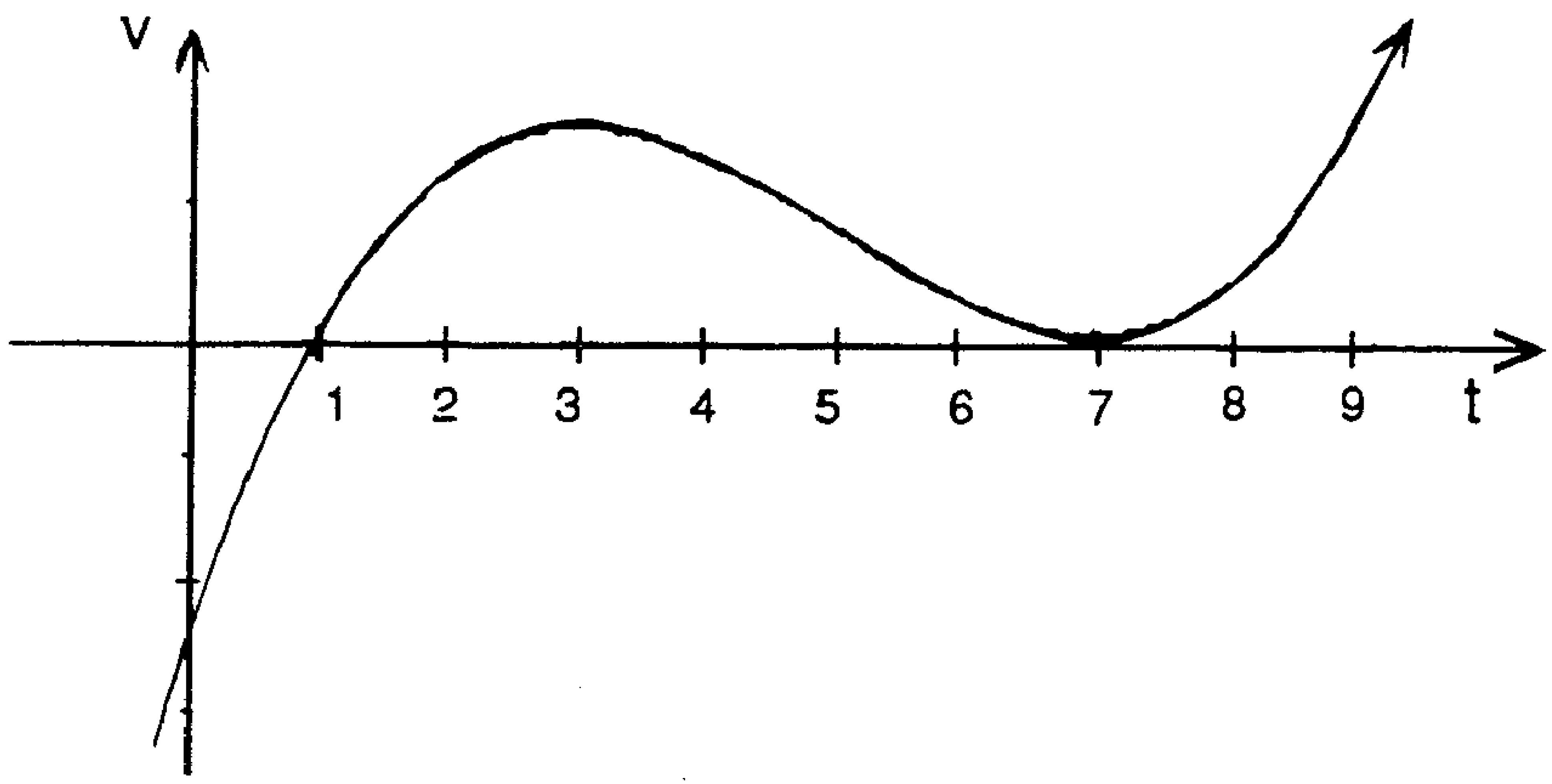
2 (a) (i) Draw a neat sketch showing the region bounded by $y = x^3$, the y-axis and the line $y = 8$.

4 (ii) Find the exact volume of the solid formed when the above region is rotated one revolution about the y-axis.

3(v) 3(x) (b) The acceleration of a particle moving along the x-axis is given by the equation $\frac{d^2x}{dt^2} = -27\cos 3t + 4e^{2t}$, where x is the displacement of the particle from the origin after time t. If when $t = 0$, $\frac{dx}{dt} = 7$ and $x = 10$ find an expression for x in terms of t.

QUESTION 8: (START A NEW PAGE)

(a) The graph drawn below represents a velocity-time graph of an object moving in a straight line.



2 (i) In which direction is the initial motion? (Give a reason for your answer)

2 (ii) At what times is the object stationary? (Give a reason for your answer)

1 (iii) When does the object change direction?

2 (iv) Sketch a possible displacement-time graph clearly showing features of this graph when $t = 1, 3$ and 7.

(b) After the engines are turned off a passenger liner continues to move forward with velocity v m/min at time t given by the formula $v = Ae^{-kt}$ where A and k are constants. If the passenger liner was travelling at 400m/min when the engines were turned off and it slowed to 50m/min in 15 minutes, find

3 (i) the values of A and k,

2 (ii) the speed of the passenger liner 30 minutes after the engines are turned off.

QUESTION 9: (START A NEW PAGE)

(a) Gas escapes from a damaged container at a rate given by $\frac{dV}{dt} = 1.2t - 360$ where V is the volume, in litres, of gas remaining in the container after t minutes. The container initially held 120000 litres of gas.

1 (i) Find the initial rate of gas leakage.

2 (ii) Find a formula for the volume of gas remaining in the container after t minutes.

2 (iii) Show that the leak is sealed before the container is empty.

(b) At a carnival, flags numbered 1, 2 ,3 ,4, ... are placed in order along a fun run course. The flags are placed 125 metres apart with the first flag 350 metres from the starting line.

2 (i) Write a formula for D_n, the distance from the starting line to the n^{th} flag.

2 (ii) Find the distance that a competitor has run if he is standing beside flag number 23.

3 (iii) If a girl runs 5 km, which is the nearest flag and how far is she from that flag?

QUESTION 10: (START A NEW PAGE)

(a) A company makes 300 chairs per month. At \$75 each they can sell all the chairs. However the price of each chair can be increased in increments of \$3 but this will result in a 4 chair reduction in sales for each \$3 increment. The company also has fixed costs of \$12 000 per month.

3 (i) If the number of \$3 increments is i, show that the monthly profit \$P is given by the formula $P = 10500 + 600i - 12i^2$.

3 (ii) Find the number of chairs that should be made and their unit price if the monthly profit is to be maximised.

(b) A pump delivers a fixed volume of water into a dam at midday each day. The water comes from a contaminated river and each delivery contains 5000 harmful bacteria. The bacteria once in the dam increase at a rate of 2% per day. Assuming the dam is not contaminated and the first delivery is at midday on January 1st,

1 (i) find the number of bacteria in the dam just after midday January 2nd,

2 (ii) show that the number of bacteria in the dam just after midday January 31st is more than 210000.

3 (iii) If fish in the dam can survive a level of 450000 bacteria, which delivery will be the lethal delivery?

THIS IS THE END OF THE PAPER

ANSWER TO 1996 - 2UNIT TRIAL

Question 1

a) 0.14

b) $e^x(2e^x - 1)$

c) 72°

d) $2x^2 + x$

e) (i) $-\frac{1}{\sqrt{2}}$

(ii) 1

f) $c = -2$

g) $x = \frac{\pi}{6}, \frac{5\pi}{6}$

Question 2

a) (i) $-3\sin 3x$

(ii) $3e^{3x+2}$

(iii) $\frac{4}{4x+3}$

b) $y = x - 1$

c) (i) $\frac{x^4}{4} + \frac{8x\sqrt{x}}{3} + C$

(ii) $\ln 3$.

Question 3

a) (i) $S(4, 9)$

(ii) —

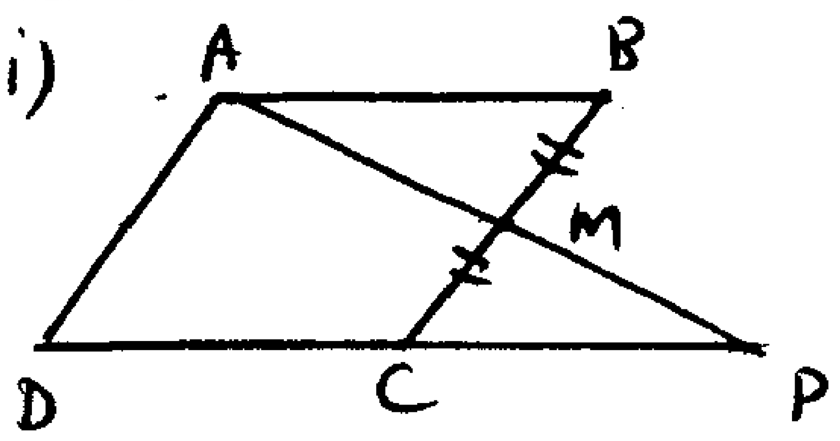
(iii) $50u^2$

b) (i) $\angle AOB = 1.6$ rads

(ii) 7.51 (2dp)

Question 4.

a) (i)



(ii) —

(iii) —

b) (i) —

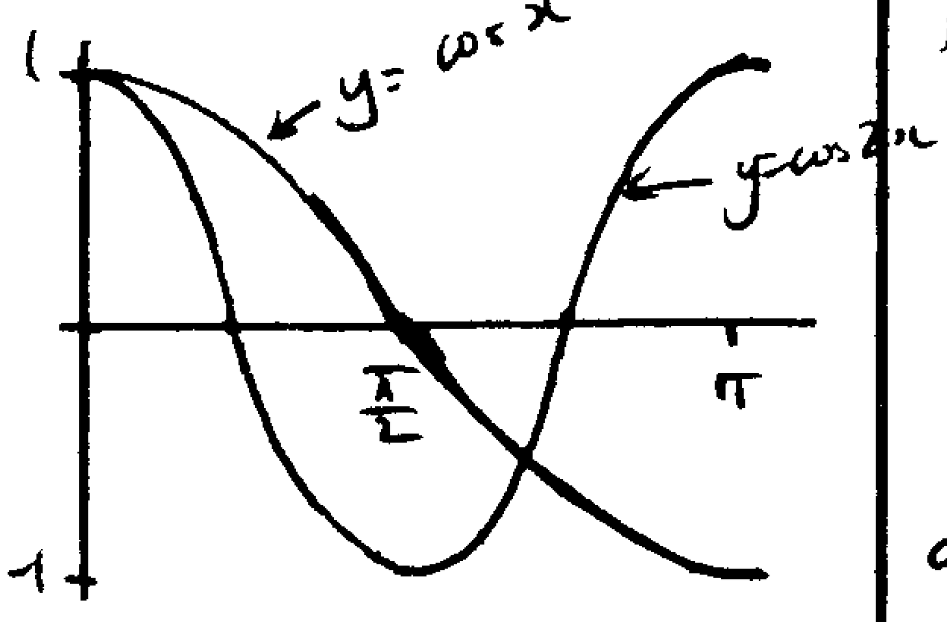
(ii) $\frac{1}{6}$

(iii) $\frac{1}{4}$

(iv) $\frac{1}{9}$

Question 5

a) (i)



(ii) —

(iii) $\frac{3\sqrt{3}}{4}$

b) (i) —

(ii) $x = 10$

Question 6

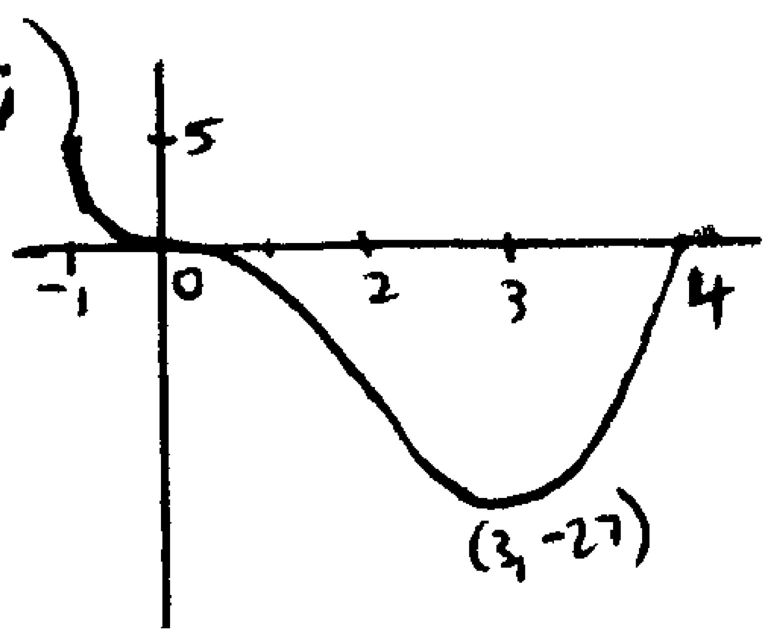
a) (i) $(0, 0)$ & $(3, -27)$

(ii) $(3, -27)$ is local min

(iii) $(0, 0)$ is a horizontal pt of inflexion

$(2, -8)$ is a pt of inflex

(iv)

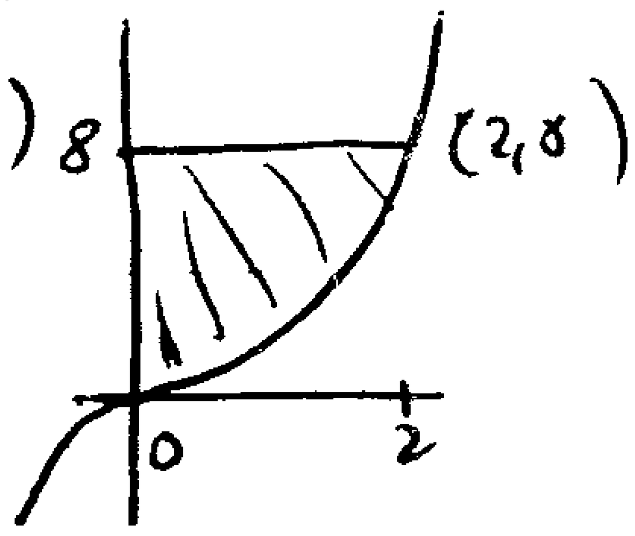


b) (i) 2800 cm^2

(ii) 21 ℓ/hr.

Qu 7.

a) (i)



(ii) $\frac{96\pi}{5}$.

b) $x = -3\cos 3t + e^{2t} + 5t + 12$

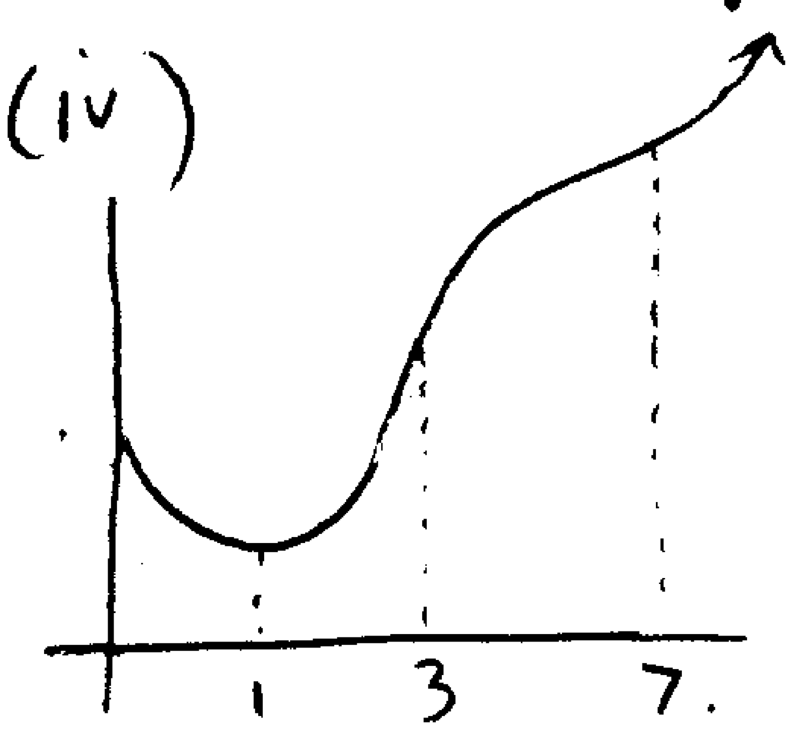
Question 8

a) (i) left, $(v < 0)$

(ii) $t = 1, 7$ $(v = 0)$

(iii) $t = 1$ (velocity changes sign)

(iv)



b) (i) $k = -\frac{1}{15}\ln\frac{1}{8}$

$A = 400$

(ii) $V = 6.25$ m/min

Question 9

a) (i) -360 ℓ/min

(ii) $V = 0.6t^2 - 360t + 120000$

(iii) —

b) (i) $D_n = 125n + 225$

(ii) 3100m

(iii) 25m

Question 10

a) (i) $P = 10500 + 600i - 12i^2$

(ii) 200, \$150 @ charr.

b) (i) 10100

(ii) —

(iii) 52nd delivery